On Intuitionistic Fuzzy R-Ideals of Semi ring

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Abstract

In this paper, we introduce the notion of intuitionistic fuzzy ideal and intuitionistic fuzzy R-ideal in semi ring and investigate some properties of intuitionistic fuzzy R-ideals of semiring.

1. Introduction:

The concept of fuzzy set μ of a set X was introduced by L. A. Zadch [9] as a function from X in [0, 1]. The concept of fuzzy ideals in a ring was introduced by W. L. Liu [8]. T. K. Dutta and B. K. Biwa's [3, 4] studied fuzzy ideals, fuzzy prime ideals of semi rings and they defined fuzzy R-ideals and fuzzy prime R-ideals of semi rings. Y. B. Jun, J. Naggers and M. S. Kim [5] extended the concept of an L-fuzzy left (resp. right) ideals of a ring to a semi ring. The concept of the idea of intuitionistic fuzzy set was first published by K. T. Atanassov [1, 2], as a generalization of the notion of fuzzy set. K.H. Kim and J. G. Lee [6] studied the intuitionistic Fuzzifications of the concept of several ideals in a semi groups and investigate some properties of such ideals. K. H. Kim [7] introduced the notion of intuitionistic Q-fuzzy semiprimality in a semi group and investigates some properties of intuitionistic Q-Fuzzifications of the concept of several ideals. In this paper we introduce the notion of intuitionistic fuzzy R-ideal of semi ring and investigate some properties of intuitionistic fuzzy R-ideal of semi rings. Throughout this paper R is a semiring.

2. Preliminaries:

Let (R, +, $\cdot$) be a smearing. By a left (right) ideal of R we mean a non-empty subset A of R such that $A + A \subseteq A$ and $RA \subseteq A$ ($AR \subseteq A$). By ideal, we mean a non-empty subset of R which both left and right ideal of R. A left ideal A of R is said to be a left R-ideal if $t \in A$, $x \in R$ and if $t + x \in A$ or $x + t \in A$ then $x \in A$. Right R-ideal is defined dually, and two sided R-ideal or simply a R-ideal is both a left and a right R-ideal. By a fuzzy set μ of a non-empty set R we mean a function $\mu: R\to [0, 1]$, and the complement of μ , denoted by $\bar{\mu}$, is the fuzzy set in R given by $\bar{\mu}$ (x)=1- μ (x), for all $x\in R$. A fuzzy set μ in R is called fuzzy left (resp. right) ideal of R if for any x, y $\in R$, $\mu(x+y)\geq \min\{\mu(x),\mu(y)\}$ and $\mu(xy) \geq \mu(y)$ ($\mu(xy) \geq \mu(x)$) and μ is called fuzzy ideal if μ both fuzzy left and right ideal of R. A fuzzy ideal μ of R is called R-fuzzy ideal of R if for any $x,y\in R$, $\mu(x)\geq \min\{\max\{\mu((x+z)+(z+y)),\mu((z+x)+(y+z))\}, \mu(y)\}$. A intuitionistic fuzzy set (IFS for short) A in a non-empty set R is an object have the form: $A = \{(x:\mu_A(x), \lambda_A(x)) / x \in R\}$ Where the function $\mu_A : R\to[0, 1]$ and $\lambda_A : R\to[0, 1]$ denoted the degree of membership and the degree of non-membership, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ An intuitionistic fuzzy set $A = \{(x : \mu_A(x), \lambda_A(x)) / x \in R\}$ in R can be identified to ordered pair (μ_A,λ_A) in $I^R \times I^R$. We shall use the symbol $A = (\mu_A, \lambda_A)$ for the IFS: $A = \{(x: \mu_A(x), \lambda_A(x)) / x\in R\}$

3. Intuitionistic fuzzy R-ideal:

Definition 3.1: An IFS $A = (\mu_A, \lambda A)$ in R is called an intuitionistic fuzzy left (resp. right) ideal of R
1- $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\mu_A(xy) \geq \mu_A(y)$ (resp. $\mu_A(xy) \geq \mu_A(x)$).
2- $\lambda_A(x+y) \leq \max\{\lambda_A(x), \lambda_A(y)\}$ and $\lambda_A(xy) \leq \lambda_A(y)$ (resp. $\lambda_A(xy) \geq \lambda_A(x)$). For all x, y $\in R$:

Definition 3.2: An intuitionistic fuzzy ideal $A= (\mu_A, \lambda_A)$ of R is called an intuitionistic fuzzy R- ideal of R if
1. $\mu_A(xy) \geq \mu_A(y)$, $\mu_A(xy) \geq \mu_A(x)$. 2 $\mu_A(x) \geq\min \{\max \{\mu_A((x+z) + (z+y)), \mu_A((z+x) +(y+z))\}, \mu_A(y)\}$ and
3. $\lambda_A(xy) \leq \lambda_A(y)$, $\lambda_A(xy) \geq \lambda_A(x)$). **4** $\lambda_A(x) \leq\max\{\min\{\lambda_A((x+z)+(z+y)),\lambda_A((z+x)+(y+z))\}, \lambda_A(y)\}.\forall$ x, $y\in R$.

Theorem 3.3: Let $A = (\mu_A, \lambda_A)$ an intuitionistic fuzzy set in R such that μ_A is fuzzy R-ideal of R then d $A = (\mu_A, \bar{\mu}_A)$ is an intuitionistic R-ideal of R.

Proof: Let x, $y\in R$, since μ_A is a fuzzy R-ideal of R $\Rightarrow$ μ_A is a fuzzy ideal.
So $\mu_A(x+y) \geq \min \{\mu_A(x), \mu A(y)\}$ and $\mu_A(xy) \geq \mu_A(y)$, $\mu_A(xy) \geq \mu_A(x)$,

$\bar{\mu}_A(x+y) =1-\mu_A(x+y) \leq 1-\min \{\mu_A(x), \mu_A(y)\} =\max \{1-\mu_A(x), 1-\mu_A(y)\} = \max \{ \bar{\mu}_A(x), \bar{\mu}_A(y)\}$

$\bar{\mu}_A(x + y) \leq \max \{ \bar{\mu}_A(x), \bar{\mu}_A(y)\}$, $\bar{\mu}_A(xy) = 1 - \mu_A(xy) \leq 1 - \mu_A(y) = \bar{\mu}_A(y)$, also $\mu_A(xy) \leq \mu_A(x)$

Therefore $dA = (\mu_A, \bar{\mu}_A)$ is an intuitionistic fuzzy ideal of R.

Let $\mu_A(x) \geq\min \{\max \{\mu_A((x+z) + (z+y)), \mu_A((z+x) +(y+z))\}, \mu_A(y)\}$
$\bar{\mu}(x)=1 - \mu(x) \geq\min \{\max \{\mu_A((x+z) + (z+y)), \mu_A((z+x) +(y+z))\}, \mu A(y)\}$,

$\overline{\mu}_A$ (x) =max {1- max {μ_A ((x+z) + (z+y)), μ_A ((z+x) +(y+z))}, μA (y)}, 1-μ_A (y)}
= max {min {1- μ_A((x+z)+(z+y)),1-μ_A((z+x)+(y+z))},1- μ_A (y)}
=max {min { $\overline{\mu}_A$ ((x+z) + (z+y)), $\overline{\mu}_A$ ((z+x)+(y+z))}, $\overline{\mu}_A$ (y)}

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so $\overline{\mu}_A$ (x)≤max {min { $\overline{\mu}_A$ ((x+z) + (z+y)), $\overline{\mu}_A$ ((z+x)+(y+z))}, $\overline{\mu}_A$ (y)} ∴ dA = (μ_A, $\overline{\mu}_A$) is an intuitionistic R-ideal

Theorem 3.4: An IFS A = (μ_A, λ_A) is an intuitionistic fuzzy R-ideal of R if and only if the fuzzy sets μ_A and $\overline{\lambda}_A$ are fuzzy R-ideals of R.

Proof: Suppose that an IFS A = (μ_A, λ_A) is an intuitionistic fuzzy R-ideal of R. Clearly μ_A is a fuzzy R-ideal.
Let x, y ∈ R. Since A = (μ_A, λ_A) is an is an intuitionistic fuzzy R-ideal.
⇒ λ_A(x + y) ≤ max {λ_A (x), λ_A (y)} and λ_A (xy) ≤ λ_A (y), λ_A (xy) ≤ λ_A(x)
λ_A (x) ≤ max {min {λ_A ((x+z) + (z+y)), λ_A ((z+x) +(y+z))}, λ_A (y)}
$\bar{\lambda}_A$ (x + y) = 1 - λ_A (x + y) ≥ 1 – max {λ_A (x), λ_A (y)} = min {$\bar{\lambda}_A$ (x), $\bar{\lambda}_A$ (y)}
So $\bar{\lambda}_A$ (x + y) ≥ min {$\bar{\lambda}_A$ (x), $\bar{\lambda}_A$ (y)}
$\bar{\lambda}_A$ (xy) = 1- λ_A (xy) ≥ 1 - λ_A (y) = $\bar{\lambda}_A$ (y),
$\bar{\lambda}_A$ (xy) ≥ λ_A (x) ≥ 1 – max $\bar{\lambda}_A$ (y). Also we can get that $\bar{\lambda}_A$ (xy) ≥ $\bar{\lambda}_A$ (x).
$\bar{\lambda}_A$ (x) = 1 – λ_A (x)
≥ 1 – max {min {λ_A ((x+z) + (z+y)), λ_A ((z+x) +(y+z))},λ_A (y)}
= max {1 – min {λ_A ((x+z) + (z+y)), λ_A ((z+x) +(y+z))}, 1 - λ_A (y)}
= min {max {1-λ_A ((x+z) + (z+y)), 1- λ_A ((z+x) +(y+z))}, $\bar{\lambda}_A$ (y)}
= min {max {$\bar{\lambda}_A$ ((x+z) + (z+y)), $\bar{\lambda}_A$ ((z+x) +(y+z))},$\bar{\lambda}_A$ (y)}
So $\bar{\lambda}_A$ (x) ≥ min {max {$\bar{\lambda}_A$ ((x+z) + (z+y)), $\bar{\lambda}_A$ ((z+x) +(y+z))}, $\bar{\lambda}_A$ (y)}
Therefore $\bar{\lambda}_A$ fuzzy R-ideals of R. suppose that μ_A and $\overline{\lambda}_A$ are fuzzy R-ideals of R
Let x, y ∈ R. Since μ_A is a fuzzy R-ideals of R.
μ_A (x +y) ≥ min {μ_A (x), μ_A(y)}
μ_A (x y) ≥ min {μ_A (x), μ_A(y)} and μ_A(x y) ≥ μ_A (y), μ_A (xy) ≥ μ_A (x)
μ_A (x) ≥ min {max {μ_A ((x+z)+(z+y)), μ_A ((z+x)+(y+z))},μ_A(y)}
λ_A(x + y) = 1 – $\bar{\lambda}_A$ (x + y). Since $\bar{\lambda}_A$ is a fuzzy ideal of R we get that
λ_A(x + y) = 1 – min { $\overline{\lambda}_A$ (x), $\bar{\lambda}_A$ (y) } = max {1-$\bar{\lambda}_A$ (x), 1-$\bar{\lambda}_A$ (y)} = max {λ_A (x), 1-λ_A (y)}
So λ_A (x + y) ≤ max {λ_A (x), λ_A (y)} Also we get that λ_A (xy) ≤ λ_A(y), λ_A (xy) ≤ λ_A(x)
λ_A (**x**) =1– $\bar{\lambda}_A$(x) Since λ_A is a fuzzy R-ideals of R we get that
λ_A(x) ≤ 1 – min {max { $\bar{\lambda}_A$ ((x+z) + (z+y)), $\bar{\lambda}_A$ ((z+x) + (y+z))}, $\bar{\lambda}_A$ (y)}
= max {1 - max { $\bar{\lambda}_A$ ((x+z) + (z+y)), $\bar{\lambda}_A$ ((z+x) + (y+z))}, 1-$\bar{\lambda}_A$ (y)}
= max {min { $\mathbf{1} - \bar{\lambda}_A$ ((x+z) + (z+y)), 1- $\bar{\lambda}_A$ ((z+x) +(y+z))}, λ_A (y)}
= max {min {λ_A ((x+z) + (z+y)), λ_A ((z+x)+(y+z))}, λ_A (y)}
So λ_A (x) ≤ max {min {λ_A ((x+z) + (z+y)), λ_A ((z+x) +(y+z))}, λ_A (y)}
Therefore= (μ_A, λ_A) is an is an intuitionistic fuzzy R-ideal

Theorem 3.7: An IFS A = (μ_A, λ_A) is an intuitionistic fuzzy R-ideal of R iff for any t ∈ [a, b] such that $(\mu_A)_t$≠ Φ and (λ_A)t ≠ Φ. $(\mu_A)_t$ and $(\lambda_A)_t$ are R-ideal of R, where (μ_A)t ={x ∈ R / μ_A(x) ≥ t}.

Proof: Suppose that IFS A = (μ_A, λ_A) is an intuitionistic fuzzy R-ideal of R So by theorem (2.3) μ_A and $\bar{\lambda}_A$ are fuzzy R-ideal of R ⇒ μ_A and λ_A are fuzzy ideal of R By [4] for any t ∈ [0, 1] such that (μ_A)t ≠ Φ and (λ_A)t ≠ Φ (μ_A)t and (λ_A)t are ideal of R. Let x ∈ $(\mu_A)_t$ and y ∈ R and ((x +z)+(z+ y))∈ (μ_A)t or ((z + x)+(y+z))∈ (μ_A)t . ⇒ μ_A(x) ≥ t and μ_A((x +z)+(z+ y))≥ t or μ_A ((z + x)+(y+z))≥t . ⇒ max {μ_A ((x +z)+(z+ y)), μ_A((z + x)+(y+z))} ≥ t. Since μ_A is fuzzy R-ideal of Rμ_A(y) ≥ mini {max {μ_A ((x +z)+(z+ y)), μ_A((z + x)+(y+z))}, μ_A(x)} ≥ t⇒μ_A(y) ≥ t so y ∈ (μ_A)t. Therefore (μ_A)t is R-ideal of R.Similarly we can prove that (λ_A)t is R-ideal of R.Suppose that for any t∈[0, 1] such (μ_A)t ≠ Φ and (λ_A)t ≠ Φ, (μ_A)t and (λ_A)t are R-ideal of R.So (μ_A)t and (λ_A)t are ideal of R.By [4] μ_A and λ_A are fuzzy ideal of R.Let x, y ∈ R and μ_A(y) = r_1 , μ_A((x +z)+(z+ y))=r_2 , μ_A((z + x)+(y+z))=r_3, (ri ∈ [0, 1])Let t= min{max{r_1, r_3}, r_1}⇒ y ∈ (μ_A)t and ((x +z)+(z+ y)) ∈ (μ_A)t or ((z + x)+(y+z))∈ (μ_A)t Since (μ_A)t is R-ideal of R.So x ∈ (μ_A)t⇒ μ_A(x)≥ t, μ_A(x)≥ min{max{μ_A((x +z)+(z+ y)),μ_A((z + x)+(y+z))},μ_A(y)}Therefore μ_A is fuzzy R-ideal of R.Similarly we can prove that λ_A is fuzzy R-ideal of R.By theorem (2.5) we get that A=(μ_A, λ_A) is an intuitionistic fuzzy R-ideal of R.Recall a function f from a semi-ring R into semi-ring T homomorphism if f (x + y) = f (x) + f (y) and f (xy) = f(x) f(y) for any x,y∈ R Let f be a function from a set X into a set Y respectively, then the image of A under f, denoted by f(A) and the preimage of B under f, denoted by f-1(B), are IFSs in X and Y respectively and defined by f (A) = (f (μ_A), f(λ_A)), f^{-1}(B)=(f^{-1} (μ_B), f^{-1} (λ_B))

Theorem 3.9: Let f: R→T be onto homomorphism of semi rings. If B= (μ_B, λ_B) is an intuitionistic fuzzy R-ideal of T then the preimage f^{-1}(B) = (f^{-1}(μ_B), f^{-1}(λ_B)) of B under f is an intuitionistic fuzzy R-ideal of R.

Proof:1- let x, y ∈ R. $f^{-1}(\mu_B)(x + y) = \mu_B(f(x + y)) = \mu_B(f(x) + f(y))$

$\geq \min\{\mu_B(f(x)), \mu_B(f(y))\} = \min\{f\text{-}1(\mu_B)(x), f\text{-}1(\mu_B)(y)\}$.

So $f^{-1}(\mu_B)(x + y) \geq \min\{f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)\}$

$f^{-1}(\mu_B)(xy) = \mu_B(f(xy)) = \mu_B(f(x) f(y))$

$\geq \mu_B(f(y)) = f^{-1}(\mu_B)(y)$.

So $f^{-1}(\mu_B)(xy) \geq f^{-1}(\mu_B)(y)$.Also $f^{-1}(\mu_B)(xy) \geq f^{-1}(\mu_B)(x)$,

$f^{-1}(\lambda_B)(x+y) = \lambda_B(f(x+y)) = \lambda_B(f(x) + f(y))$

$\leq \max\{\lambda_B(f(x)), \lambda_B(f(y))\}$

$= \max\{f^{-1}(\lambda_B)(x), f^{-1}(\lambda_B)(y)\}$

So $f^{-1}(\lambda B)(x+y) \leq \max\{f^{-1}(\lambda_B)(x), f^{-1}(\lambda_B)(y)\}$.

$f^{-1}(\lambda_B)(xy) = \lambda_B(f(xy)) = \lambda_B(f(x) f(y))$

$\leq \lambda_B(f(y)) = f^{-1}(\lambda_B)(y)$.

So $f^{-1}(\lambda_B)(xy) \geq f^{-1}(\lambda_B)(y)$.Also $f^{-1}(\lambda_B)(xy) \geq f^{-1}(\lambda_B)(x)$

2- Let x, y ∈ R⇒ f(x), f(y) ∈ T.

$f^{-1}(\mu_B)(x) \geq \mu_B(f(x)) \geq \min\{\max\{\mu_B(f(x+z) + f(z+y)), \mu B(f(z+x) + f(y+z))\}, \mu_B(f(y))\}$

$= \min\{\max\{f^{-1}(\mu_B)((x+z)+(z+y)), f^{-1}(\mu_B)((z+x)+(y+z))\}, f^{-1}(\mu_B)(y)\}$

$f^{-1}(\lambda_B)(x) = \lambda_B(f(x)) \leq \max\{\min\{\lambda_B(f(x+z) + f(z+y)), \lambda_B(f(z+x) + f(y+z))\}, \lambda_B(f(y))\}$

$= \max\{\min\{f^{-1}(\lambda_B)((x+z)+(z+y)), f^{-1}(\lambda_B)((z+x)+(y+z))\}, f^{-1}(\lambda_B)(y)\}$

Therefore f^{-1} (B) = (f^{-1} (μ_B), f^{-1} (λ_B)) is an intuitionistic fuzzy R-ideal of R.

Theorem 5. If μ_A is a intuitionistic fuzzy R- ideal in semi ring R then μ_A^m is also intuitionistic fuzzy R- ideal in semi ring R.

Proof: For all x, y ∈ R.

$$\{\mu_A(x)\}^m \geq \left\{min\left\{max\left\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\right\}, \mu_A(y)\right\}\right\}^m$$

$$\mu_A(x)^m \geq min\left\{max\left\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\right\}, \mu_A(y)\right\}^m$$

$$\mu_A^m(x) \geq min\left\{max\left\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\right\}^m, \mu_A(y)^m\right\}$$

$$\geq min\left\{max\left\{\mu_A((x+z)+(z+y))^m, \mu_A((z+x)+(y+z))^m\right\}, \mu_A(y)^m\right\}$$

$$\mu_A^m(x) \geq min\left\{max\left\{\mu_A^m((x+z)+(z+y)), \mu_A^m((z+x)+(y+z))\right\}, \mu_A^m(y)\right\}$$

$$\{\lambda_A(x)\}^m \leq \left\{\max\left\{\min\left\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\right\}, \lambda_A(y)\right\}\right\}^m$$

$$\lambda_A(x)^m \leq \max\left\{\min\left\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\right\}, \lambda_A(y)\right\}^m$$

$$\lambda_A^m(x) \leq \max\left\{\min\left\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\right\}^m, \lambda_A(y)^m\right\}$$

$$\leq \max\left\{\min\left\{\lambda_A((x+z)+(z+y))^m, \lambda_A((z+x)+(y+z))^m\right\}, \lambda_A(y)^m\right\}$$

$$\lambda_A^m(x) \leq \max\left\{\min\left\{\lambda_A^m((x+z)+(z+y)), \lambda_A^m((z+x)+(y+z))\right\}, \lambda_A^m(y)\right\}$$

Therefore μ_A^m is a fuzzy ideal in semi ring R

Theorem 7: If μ_A is a fuzzy ideal in semi ring R then $\mu_{A\cap B}$ is also fuzzy ideal in semi ring R.

Proof: For all x, y ∈ R. $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$ and $\mu_A(xy) \geq \mu_A(y)$, $\mu_A(xy) \geq \mu_A(x)$

$\mu_B(x+y) \geq \min\{\mu_B(x), \mu_B(y)\}$ and $\mu_B(xy) \geq \mu_B(y)$, $\mu_B(xy) \geq \mu_B(x)$

$$\min\{\mu_A(x+y), \mu_B(x+y)\} \ge \min\{\min\{\mu_A(x)+\mu_A(y)\}, \min\{\mu_B(x)+\mu_B(y)\}\}$$
$$\min\{\mu_A(xy), \mu_B(xy)\} \ge \min\{\mu_A(y), \mu_B(y)\}, \min\{\mu_A(xy), \mu_B(xy)\} \ge \min\{\mu_A(x), \mu_B(x)\}$$
$$\mu_{A\cap B}(x+y) \ge \min\{\min\{\mu_A(x), \mu_B(x)\}, \{\min\{\mu_B(x), \mu_B(y)\}\} = \min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$$
$$\mu_{A\cap B}(xy) \ge \mu_{A\cap B}(y),\ \mu_{A\cap B}(xy) \ge \mu_{A\cap B}(x).$$

Theorem 8. If μ_A is a fuzzy R- ideal in semi ring R then $\mu_{A\cap B}$ is also fuzzy R- ideal in semi ring R.

Proof: For all x, y ∈ R. ∀x, y∈R. Now,

$$\mu_A(x) \ge \min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\}$$
$$\mu_B(x) \ge \min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}$$
$$\min\{\mu_A(x), \mu_B(x)\} \ge \min\{\min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\},$$
$$\min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}\}$$

If one is contained in another.

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$$\mu_{A\cap B}(x) \ge \min\{\max\{\min\{\mu_A((x+z)+(z+y)), \mu_B((x+z)+(z+y))\},$$
$$\{\mu_A((z+x)+(y+z)), \mu_B((z+x)+(y+z))\}\}, \max\{\min\{\mu_A(y), \mu_B(y)\}\}\}$$
$$\mu_{A\cap B}(x) \ge \min\{\max\{\mu_{A\cap B}((x+z)+(z+y)),\ \mu_{A\cap B}((z+x)+(y+z))\}, \mu_{A\cap B}(y)\}$$

Theorem 9: If μ_A is a fuzzy ideal in semi ring R then $\mu_{A\cup B}$ is also fuzzy ideal in semi ring R.

Proof: For all x, y ∈ R. μ_A (x+y) ≥ min {μ_A (x), μ_A (y)} and μ_A (xy) ≥ μ_A (y), μ_A (xy) ≥ μ_A (x)
μ_B (x+y) ≥ min{μ_B (x), μ_B (y)} and μ_B (xy) ≥ μ_B (y), μ_B (xy) ≥ μ_B (x)

$$\max\{\mu_A(x+y), \mu_B(x+y)\} \ge \max\{\min\{\mu_A(x)+\mu_A(y)\}, \min\{\mu_B(x)+\mu_B(y)\}\}$$

If one is contained in another.

$$\max\{\mu_A(xy), \mu_B(xy)\} \ge \max\{\mu_A(y), \mu_B(y)\}, \max\{\mu_A(xy), \mu_B(xy)\} \ge \max\{\mu_A(x), \mu_B(x)\}$$
$$\mu_{A\cup B}(x+y) \ge \min\{\max\{\mu_A(x), \mu_B(x)\}, \{\max\{\mu_B(x), \mu_B(y)\}\} = \min\{\mu_{A\cup B}(x), \mu_{A\cup B}(y)\}$$
$$\mu_{A\cup B}(xy) \ge \mu_{A\cup B}(y),\ \mu_{A\cup B}(xy) \ge \mu_{A\cup B}(x).$$

Theorem10. If μ_A is a fuzzy R- ideal in semi ring R then $\mu_{A\cup B}$ is also fuzzy R- ideal in semi ring R.

Proof: For all x, y ∈ R. ∀x, y∈R. Now,

$$\mu_A(x) \ge \min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\}$$
$$\mu_B(x) \ge \min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}$$
$$\max\{\mu_A(x), \mu_B(x)\} \ge \max\{\min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\},$$
$$\min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}\}$$

If one is contained in another.

$$\mu_{A\cup B}(x) \ge \min\{\max\{\max\{\mu_A((x+z)+(z+y)), \mu_B((x+z)+(z+y)),$$
$$\{\mu_A((z+x)+(y+z)), \mu_B((z+x)+(y+z))\}\}, \max\{\max\{\mu_A(y), \mu_B(y)\}\}$$
$$\mu_{A\cup B}(x) \ge \min\{\max\{\mu_{A\cup B}((x+z)+(z+y)),\ \{\mu_{A\cup B}((z+x)+(y+z))\}\}, \mu_{A\cup B}(y)\}$$

Theorem 11. If μ_A **is** an intuitionistic fuzzy ideal of semi ring R then $\mu_{A\cap B}$ is an intuitionistic fuzzy ideal of semi ring R of one is contains another.

Proof: If μ_A is a fuzzy ideal in semi ring R then $\mu_{A\cap B}$ is also fuzzy ideal in semi ring R.

Proof: For all x, y ∈ R. μ_A (x+y) ≥ min {μ_A (x), μ_A (y)} and μ_A (xy) ≥ μ_A (y), μ_A (xy) ≥ μ_A (x)
μ_B (x+y) ≥ min{μ_B (x), μ_B (y)} and μ_B (xy) ≥ μ_B (y), μ_B (xy) ≥ μ_B (x)

$$\min\{\mu_A(x+y), \mu_B(x+y)\} \ge \min\{\min\{\mu_A(x),\ \mu_A(y)\}, \min\{\mu_B(x), \mu_B(y)\}\}$$
$$\mu_{A\cap B}(x+y) \ge \min\{\min\{\mu_A(x), \mu_B(x)\}, \{\min\{\mu_B(x), \mu_B(y)\}\}$$
$$\mu_{A\cap B}(x+y) \ge \min\{\mu_{A\cap B}(x), \mu_{A\cap B}(y)\}$$
$$\min\{\mu_A(xy), \mu_B(xy)\} \ge \min\{\mu_A(y), \mu_B(y)\}, \textit{and} \min\{\mu_A(xy), \mu_B(xy)\} \ge \min\{\mu_A(x), \mu_B(x)\}$$
$$\mu_{A\cap B}(xy) \ge \mu_{A\cap B}(y),\ \mu_{A\cap B}(xy) \ge \mu_{A\cap B}(x).$$

Now ∀x, y∈R, now

λ_A (x+y) ≥ min {λ_A (x), λ_A(y)},λ_A (xy) ≥ λ_A (y), λ_A (xy) ≥ λ_A (x)

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λ_B (x+y) ≥ min {λ_B (x), λ_B (y)} and λ_B (xy) ≥ λ_B (y), λ_B (xy) ≥ λ_B (x)

$\min\{\lambda_A(x+y), \lambda_B(x+y)\} \geq \min\{\max\{\lambda_A(x), \lambda_A(y)\}, \max\{\lambda_B(x)+\lambda_B(y)\}\}$

$\lambda_{A\cap B}(x+y) \geq \max\{\min\{\lambda_A(x), \lambda_B(x)\}, \{\min\{\lambda_B(x), \lambda_B(y)\}\}$

$\lambda_{A\cap B}(x+y) \geq \max\{\min\{\lambda_A(x), \lambda_B(x)\}, \{\min\{\lambda_A(y), \lambda_B(y)\}\}$

$\lambda_{A\cap B}(x+y) \geq \max\{\lambda_{A\cap B}(x), \lambda_{A\cap B}(y)\}$

Theorem 12if μ_A is an intuitionistic fuzzy- R ideal of semi ring R then $\mu_{A\cap B}$ is an intuitionistic fuzzy –R ideal of
semi ring R if one is contains another.
proof: for all x, y,∈ R

$\mu_A(x) \geq \min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\}$

$\mu_B(x) \geq \min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}$ and

$\mu_A(xy) \geq \mu_A(y), \mu_B(xy) \geq \mu_B(y)$

$\min\{\mu_A(x), \mu_B(x)\} \geq \min\{\min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\},$

$\min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}\}$ and,

$\min\{\mu_A(xy), \mu_A(xy)\} \geq \min\{\mu_A(y), \mu_B(y)\}$

$\mu_{A\cap B}(x) \geq \min\{\max\{\min\{\mu_A((x+z)+(z+y)), \mu_B((x+z)+(z+y)), \mu_A((z+x)+(y+z)),$

$\mu_B((z+x)+(y+z))\}\}, \min\{\mu_A(y), \mu_B(y)\}\}$ and $\mu_{A\cap B}(xy) \geq \mu_{A\cap B}(y)$

$\mu_{A\cap B}(x) \geq \min\{\max\{\mu_{A\cap B}((x+z)+(z+y)), \mu_{A\cap B}((x+z)+(z+y))\}, \mu_{A\cap B}(y)\},$

and $\mu_{A\cap B}(xy) \geq \mu_{A\cap B}(y)$

$\lambda_A(x) \leq \max\{\min\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\}, \lambda_A(y)\}$

$\lambda_B(x) \leq \max\{\min\{\lambda_B((x+z)+(z+y)), \lambda_B((z+x)+(y+z))\}, \lambda_B(y)\}$ and

$\lambda_A(xy) \leq \lambda_A(y), \lambda_B(xy) \leq \lambda_B(y)$

$\max\{\lambda_A(x), \lambda_B(x)\} \leq \max\{\max\{\min\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\}, \lambda_A(y)\},$

$\max\{\min\{\lambda_B((x+z)+(z+y)), \lambda_B((z+x)+(y+z))\}, \lambda_B(y)\}\}$ and,

$\max\{\lambda_A(xy), \lambda_B(xy)\} \leq \max\{\lambda_A(y), \lambda_B(y)\}$

$\lambda_{A\cap B}(x) \leq \max\{\min\{\max\{\lambda_A((x+z)+(z+y)), \lambda_B((x+z)+(z+y)), \lambda_A((z+x)+(y+z)),$

$\lambda_B((z+x)+(y+z))\}\}, \max\{\lambda_A(y), \lambda_B(y)\}\}$ and $\lambda_{A\cap B}(xy) \leq \lambda_{A\cap B}(y)$

$\lambda_{A\cap B}(x) \leq \max\{\min\{\lambda_{A\cap B}((x+z)+(z+y)), \lambda_{A\cap B}((x+z)+(z+y))\}, \lambda_{A\cap B}(y)\},$

and $\lambda_{A\cap B}(xy) \leq \lambda_{A\cap B}(y)$

13 if μ_A **is** an intuitionistic fuzzy- R ideal of semi ring R then $\mu_{A\cup B}$ is an intuitionistic fuzzy –R ideal of semi ring R if one is contains another.

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$\mu_A(x) \geq \min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\}$

$\mu_B(x) \geq \min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}$ and

$\mu_A(xy) \geq \mu_A(y), \mu_B(xy) \geq \mu_B(y)$

$\max\{\mu_A(x), \mu_B(x)\} \geq \max\{\min\{\max\{\mu_A((x+z)+(z+y)), \mu_A((z+x)+(y+z))\}, \mu_A(y)\},$

$\min\{\max\{\mu_B((x+z)+(z+y)), \mu_B((z+x)+(y+z))\}, \mu_B(y)\}\}$ and,

$\max\{\mu_A(xy), \mu_A(xy)\} \geq \max\{\mu_A(y), \mu_B(y)\}$

$\mu_{A\cup B}(x) \geq \max\{\min\{\max\{\mu_A((x+z)+(z+y)), \mu_B((x+z)+(z+y)), \mu_A((z+x)+(y+z)),$

$\mu_B((z+x)+(y+z))\}\}, \max\{\mu_A(y), \mu_B(y)\}\}$ and $\mu_{A\cup B}(xy) \geq \mu_{A\cup B}(y)$

$\mu_{A\cup B}(x) \geq \min\{\max\{\mu_{A\cup B}((x+z)+(z+y)), \mu_{A\cup B}((x+z)+(z+y))\}, \mu_{A\cup B}(y)\},$

and $\mu_{A\cup B}(xy) \geq \mu_{A\cup B}(y)$

$\lambda_A(x) \leq \max\{\min\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\}, \lambda_A(y)\}$

$\lambda_B(x) \leq \max\{\min\{\lambda_B((x+z)+(z+y)), \lambda_B((z+x)+(y+z))\}, \lambda_B(y)\}$ and

$\lambda_A(xy) \leq \lambda_A(y), \lambda_B(xy) \leq \lambda_B(y)$

$\max\{\lambda_A(x), \lambda_B(x)\} \leq \max\{\max\{\min\{\lambda_A((x+z)+(z+y)), \lambda_A((z+x)+(y+z))\}, \lambda_A(y)\},$

$\max\{\min\{\lambda_B((x+z)+(z+y)), \lambda_B((z+x)+(y+z))\}, \lambda_B(y)\}\}$ and,

$\max\{\lambda_A(xy), \lambda_B(xy)\} \leq \max\{\lambda_A(y), \lambda_B(y)\}$

$\lambda_{A\cap B}(x) \leq \max\{\min\{\max\{\lambda_A((x+z)+(z+y)), \lambda_B((x+z)+(z+y)), \lambda_A((z+x)+(y+z)),$

$\lambda_B((z+x)+(y+z))\}\}, \max\{\lambda_A(y), \lambda_B(y)\}\}$ and $\lambda_{A\cap B}(xy) \leq \lambda_{A\cap B}(y)$

$\lambda_{A\cap B}(x) \leq \max\{\min\{\lambda_{A\cap B}((x+z)+(z+y)), \lambda_{A\cap B}((x+z)+(z+y))\}, \lambda_{A\cap B}(y)\},$

and $\lambda_{A\cap B}(xy) \leq \lambda_{A\cap B}(y)$

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